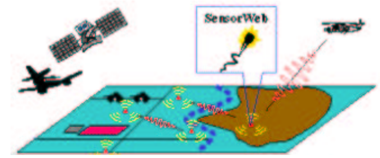

Optimum Signaling Strategies in Low-Power Networks

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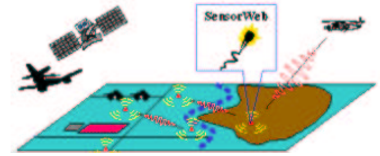
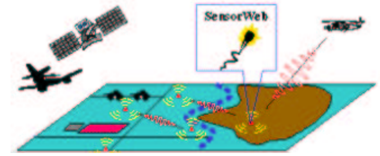
SensorWeb MURI Review Meeting



Overview

- Minimum Energy per Bit
- Power-Bandwidth Tradeoff
- Multiple Access Channels
- Broadcast Channels
- Relay Channels
- Impact of Delay Constraints



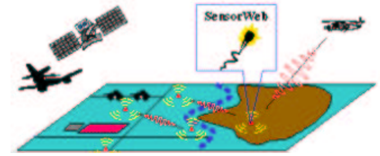
Minimum Energy per bit

SV: "On channel capacity per unit cost," *IEEE Trans. Information Theory*, vol. 36 (5), pp. 1019–1030, Sep. 1990.

$$\frac{E_b}{N_0} = \frac{\log_e 2}{\dot{C}(0)} \tag{1}$$

$$\dot{C}(0) = \sup_{x \in \mathcal{A}} \frac{D(P_{Y|X=x} || P_{Y|X=0})}{b[x]} \tag{2}$$

Achieved by **On-Off Keying**



Fading Channels

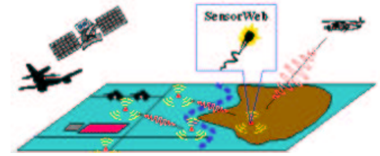
Theorem 1 *(SV, 2002) Consider the m-dimensional complex channel*

$$\mathbf{y} = \mathbf{H}\mathbf{x} + \mathbf{n} \tag{3}$$

where the complex Gaussian vector $\mathbf{n}$ *has independent identically distributed components. Then, the required received energy per bit for reliable communication satisfies*

$$\frac{E_b^{\mathsf{r}}}{N_0}_{\min} = \log_e 2 = -1.59\ dB, \tag{4}$$

regardless of whether $\mathbf{H}$ *is known at the transmitter and/or receiver.*



Traditional Optimality Criterion in low-power regime:

Input Signal is optimal in the low-power regime

$$\Leftrightarrow$$

it achieves $\frac{E_b}{N_0}_{\min}$

$$\Leftrightarrow$$

$$\lim_{\mathsf{SNR}\downarrow 0} \frac{I(\mathbf{x}_{\mathsf{SNR}};\mathbf{y})}{\mathsf{SNR}} = \dot{C}(0).$$

BPSK asymp. Optimal for Quadrature Channel?

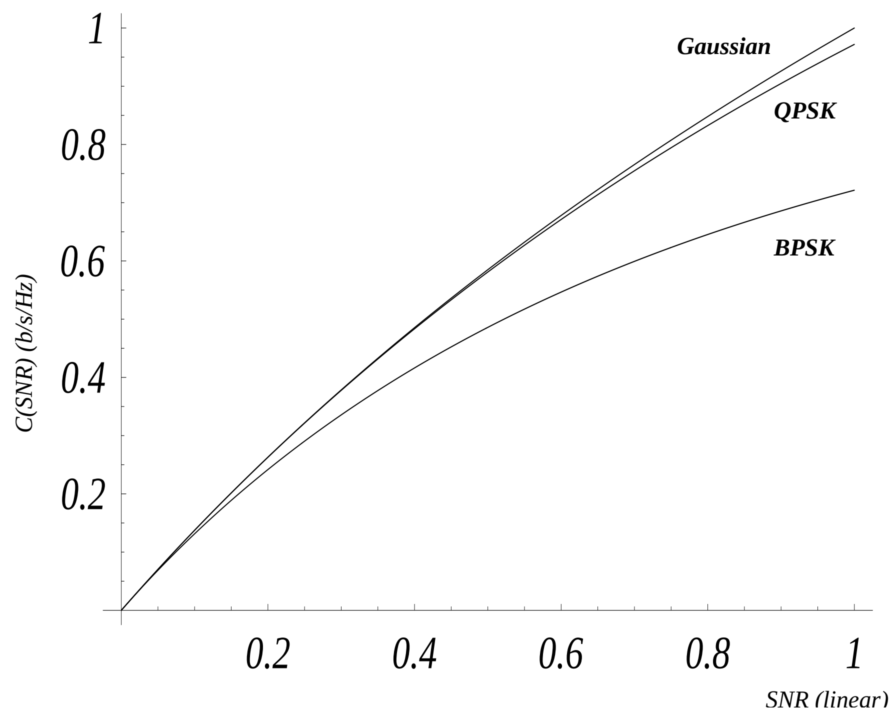
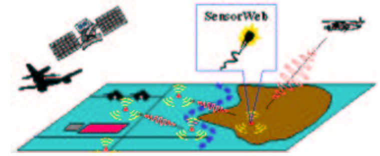


Figure 1: Capacity achieved by complex Gaussian inputs, QPSK and BPSK in the additive white Gaussian noise channel.



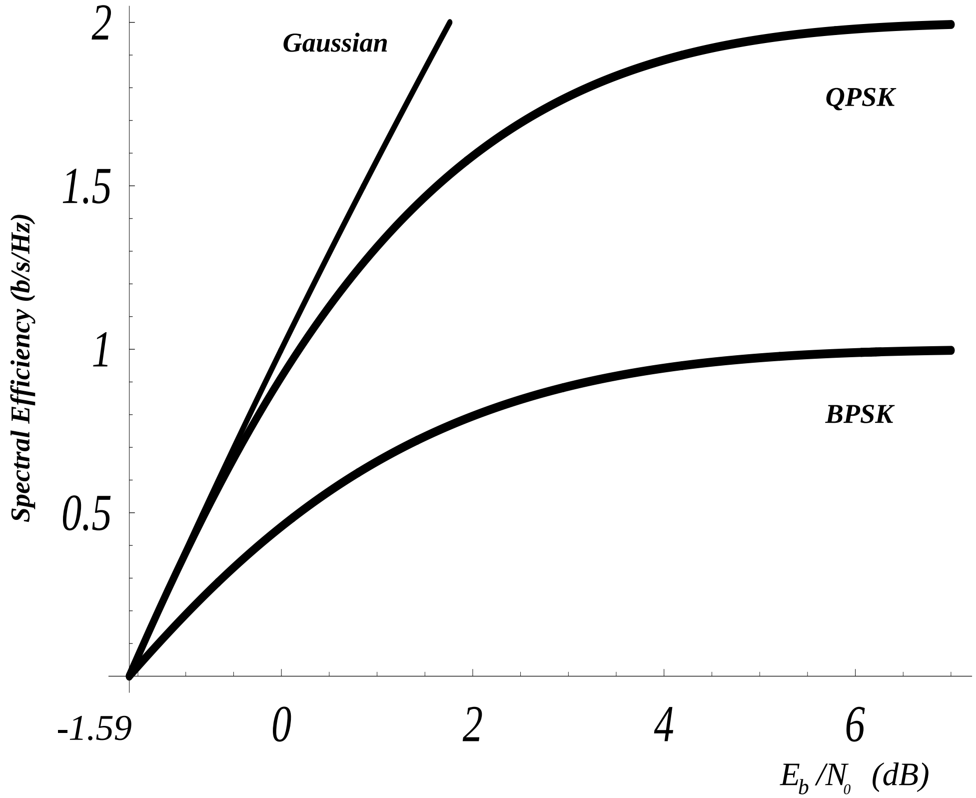


Figure 2: Spectral efficiencies achieved by complex Gaussian inputs, QPSK and BPSK in the additive white Gaussian noise channel.

New Optimality Criterion in low-power regime (SV, 2002) ::

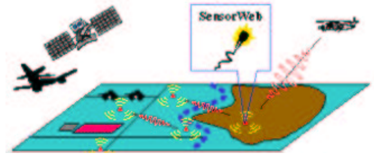
$$\text{Achieve } \dot{C}(0) \text{ and } \ddot{C}(0)$$

$$\Leftrightarrow$$

Achieve $\frac{E_b}{N_0}_{\min}$ and the optimum slope (b/s/Hz/(3 dB)of capacity at $\frac{E_b}{N_0}_{\min}$

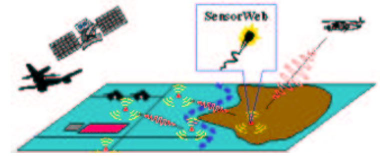
$$\Leftrightarrow$$

Minimize required bandwidth for given power and rate in the low power regime.



Optimal Power-Bandwidth Strategies::

- Coherent communication: **QPSK**
  On-Off keying requires more than 6 times as much bandwidth
- Noncoherent communication: **Flash Signaling**



The Multiple Access Channel

$$Y = X_1 + X_2 + \ldots + X_K + N \tag{5}$$

where N is Gaussian with independent real and imaginary components and $E[|N|^2] = \sigma^2$, $E[|X_i|^2] \leq P_i$.

The total capacity (maximum sum of rates) of the multiaccess channel is equal to the capacity of a single-user channel whose power is equal to the sum of the individual powers, namely

$$\log_2 \left(1 + \frac{\sum_{k=1}^{K} P_k}{\sigma^2} \right).$$

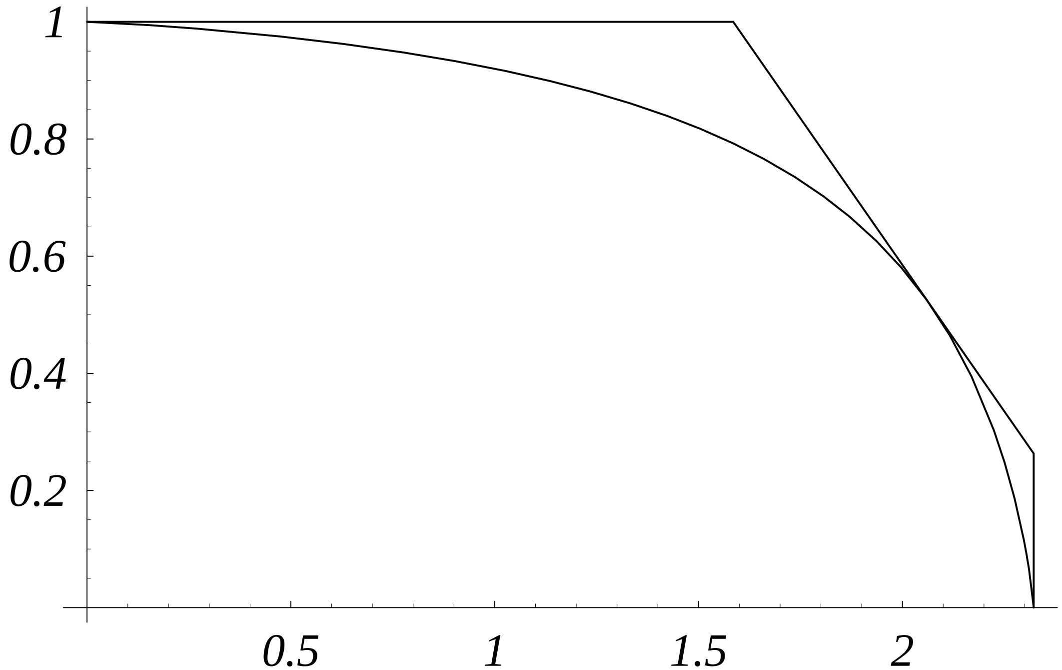
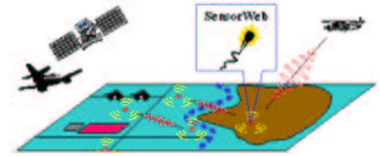


Figure 3: Multiaccess channel capacity region and TDMA achievable region with with $P_1/\sigma^2 = 4$ and $P_2/\sigma^2 = 1$.



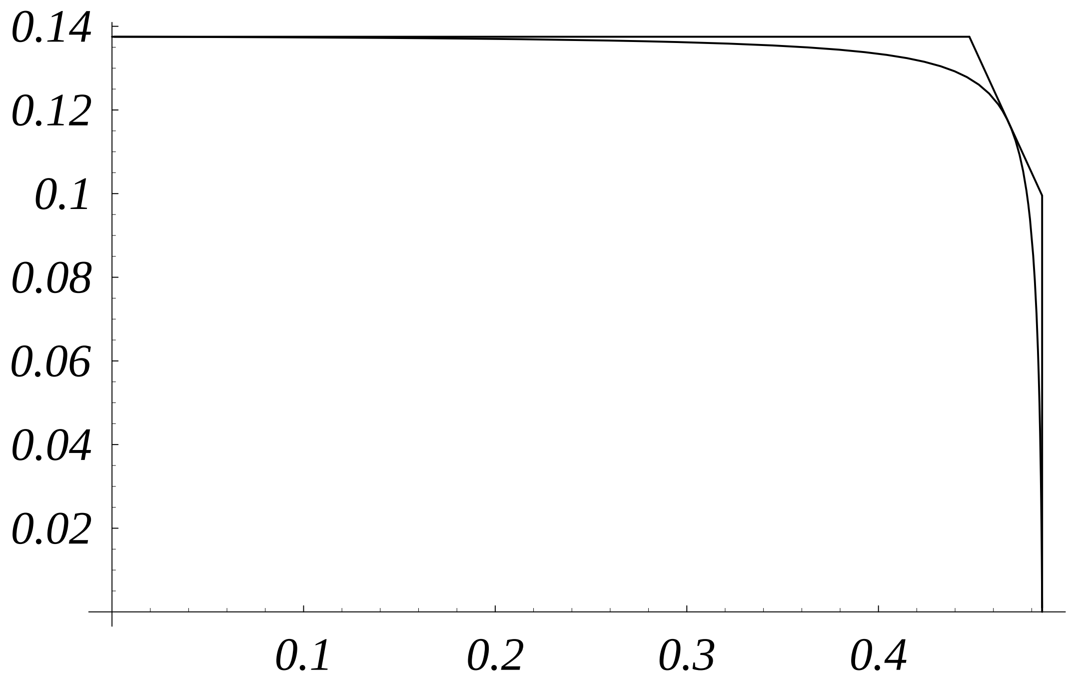
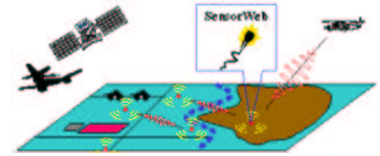


Figure 4: Multiaccess channel capacity region and TDMA achievable region with with $P_1/\sigma^2 = 0.4$ and $P_2/\sigma^2 = 0.1$.

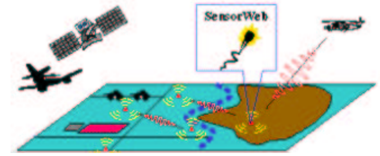


Multiaccess Channels: Optimality of TDMA

Theorem 2 *For all R_1/R_2, the minimum energies per information bit for the multiple-access channel are equal to*

$$\frac{E_1}{N_0} = \frac{E_2}{N_0} = \log_e 2 = -1.59dB. \tag{6}$$

Furthermore, (6) is achieved by TDMA.



Multiaccess Channels: Suboptimality of TDMA

Theorem 3 *For all R_1/R_2, the multiaccess slope region achieved by TDMA is:*

$$\{(\mathcal{S}_1, \mathcal{S}_2) : \ 0 \leq \mathcal{S}_1, \ 0 \leq \mathcal{S}_2, \ \mathcal{S}_1 + \mathcal{S}_2 \leq 2\}.$$

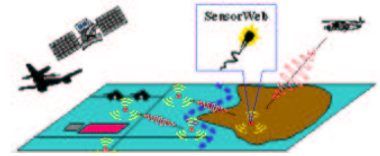
Theorem 4 *The optimum multiaccess slope region (achieved by superposition) is:*

$$\{(\mathcal{S}_1, \mathcal{S}_2) : \ 0 \leq \mathcal{S}_1 \leq 2, \ 0 \leq \mathcal{S}_2 \leq 2\}.$$

Broadcast Channels

$$
\begin{aligned}
Y_1 &= X + N_1 \\
Y_2 &= X + N_2 \\
&\cdots \\
Y_K &= X + N_K
\end{aligned}
\tag{7}
$$

where $E[|X|^2] \leq P$, $E[|N_i|^2] \leq \sigma_i^2$. The capacity region of this channel (achieved by superposition and successive cancellation) was found in (Cover' 73)



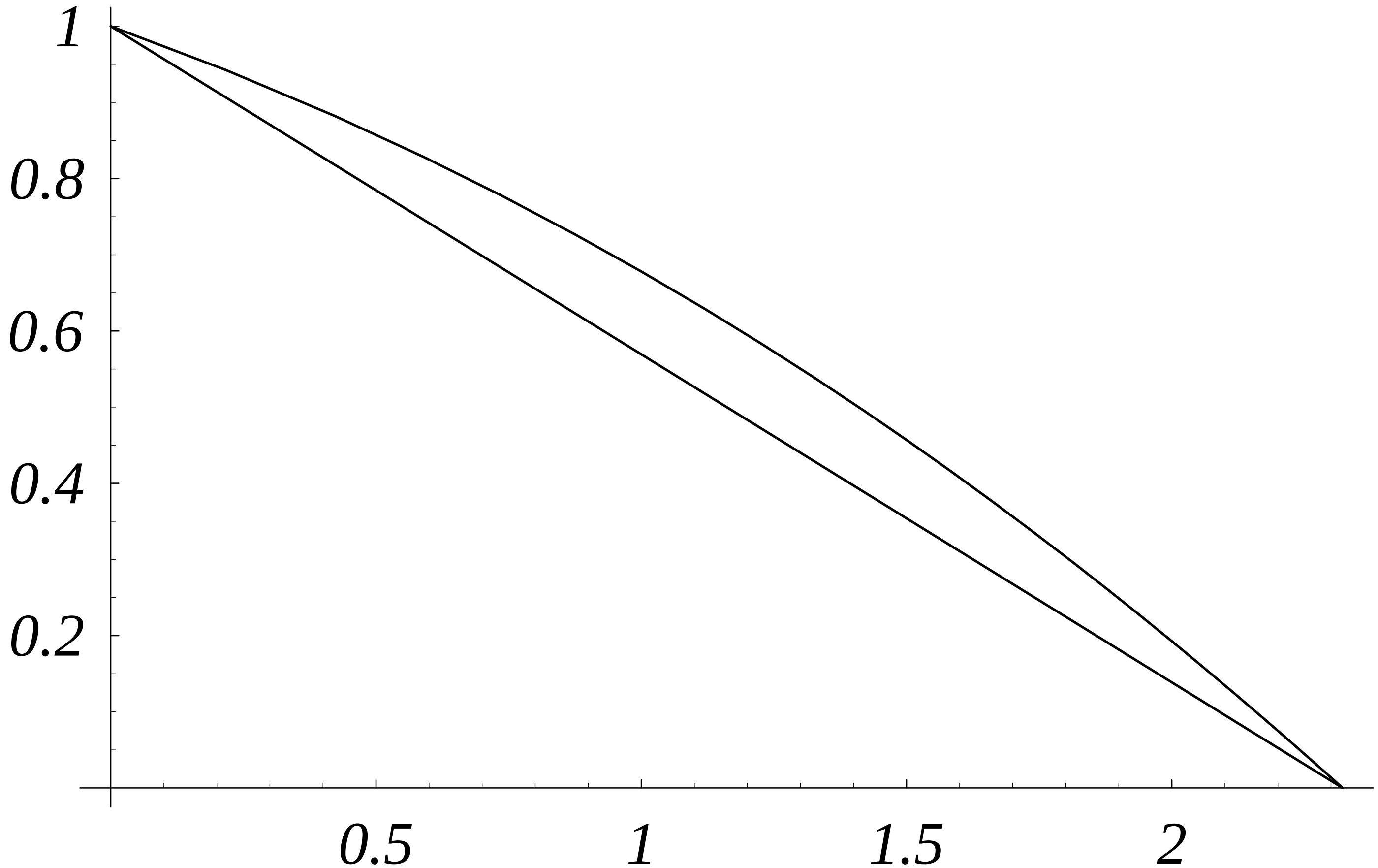


Figure 5: Capacity region and TDMA-achievable rate region of broadcast channel with $P/\sigma_1^2 = 4$ and $P/\sigma_2^2 = 1$.

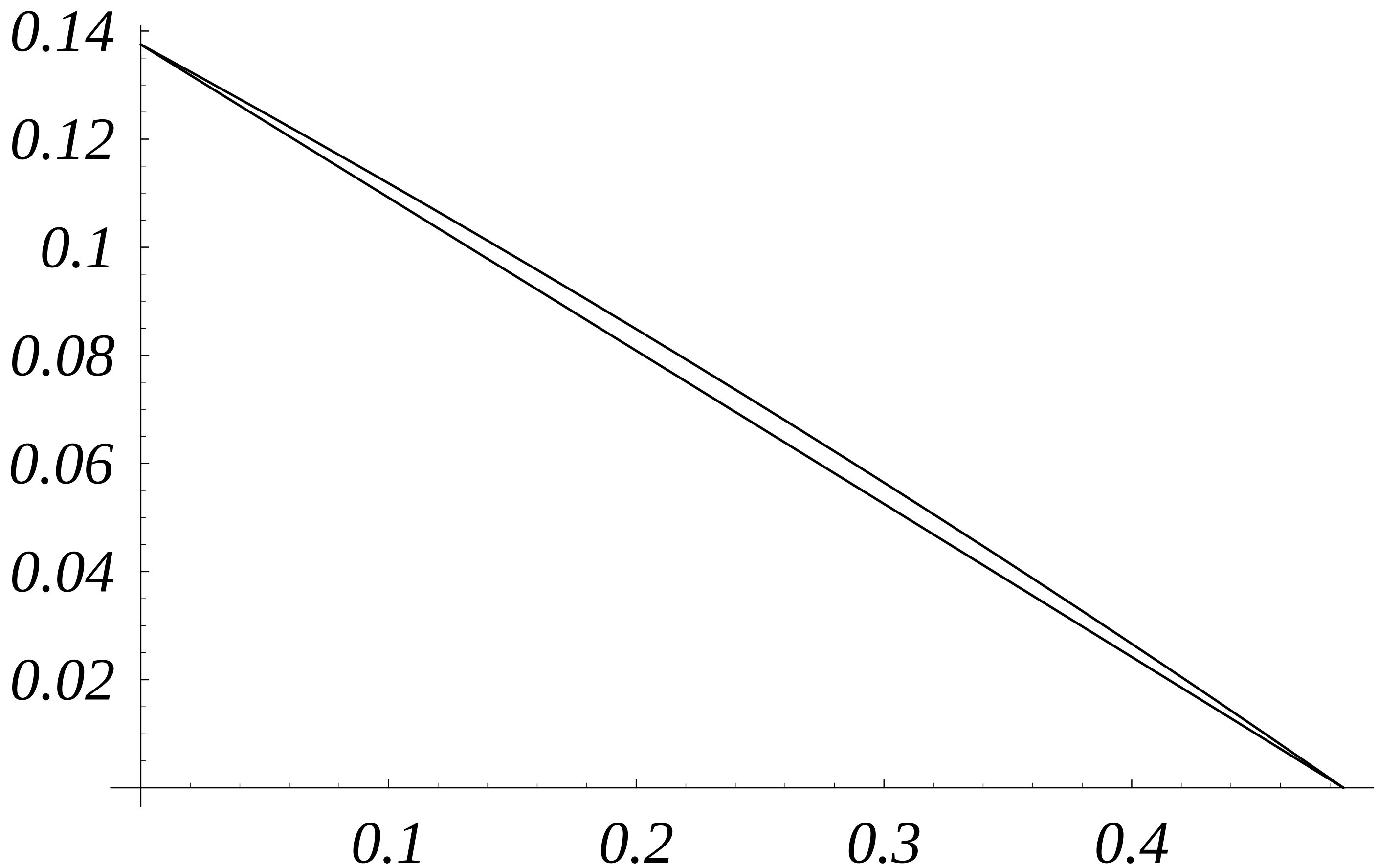


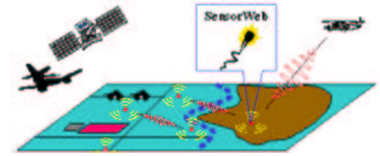
Figure 6: Capacity region and TDMA-achievable rate region of broadcast channel with $P/\sigma_1^2 = 0.4$ and $P/\sigma_2^2 = 0.1$.

Broadcast: Optimality of TDMA

Theorem 5 *Suppose that $R_1/R_2 = \theta$. Then, the minimum energies per bit achieved by both TDMA and superposition are:*

$$\frac{E_1}{N_0} = \left(1 + \frac{\sigma_2^2}{\sigma_1^2 \theta}\right) \log_e 2 \tag{8}$$

$$\frac{E_2}{N_0} = \left(1 + \frac{\theta \sigma_1^2}{\sigma_2^2}\right) \log_e 2 \tag{9}$$



Broadcast: Suboptimality of TDMA

Theorem 6 *(SV, 2002) Let the rates vanish while keeping $R_1/R_2 = \theta$. The broadcast slope region achieved by TDMA is:*

$$\{(\mathcal{S}_1, \mathcal{S}_2):\ 0 \leq \mathcal{S}_1 \leq \frac{2\theta}{1+\theta},\ 0 \leq \mathcal{S}_2 \leq \frac{2}{1+\theta}\}. \tag{10}$$

Theorem 7 *(SV, 2002) Let the rates vanish while keeping $R_1/R_2 = \theta$. The optimum broadcast slope region (achieved by superposition) is:*

$$\begin{aligned} \{(\mathcal{S}_1, \mathcal{S}_2):\ 0 &\leq \mathcal{S}_1 \leq \frac{2\theta\left(\theta + \sigma_2^2/\sigma_1^2\right)}{\theta^2 + 2\theta + \sigma_2^2/\sigma_1^2}, \\ 0 &\leq \mathcal{S}_2 \leq \frac{2\left(\theta + \sigma_2^2/\sigma_1^2\right)}{\theta^2 + 2\theta + \sigma_2^2/\sigma_1^2}\}. \end{aligned} \tag{11}$$

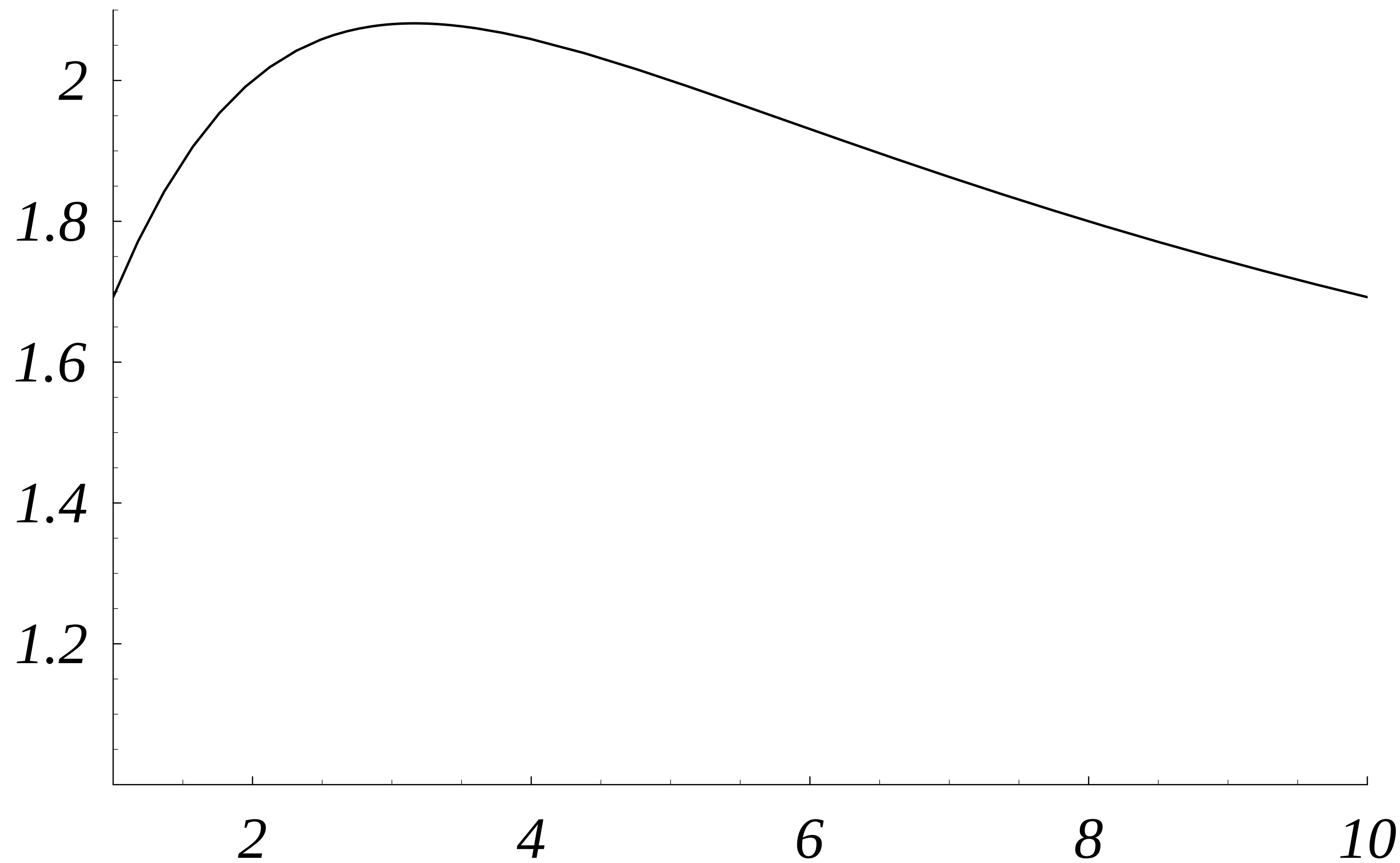
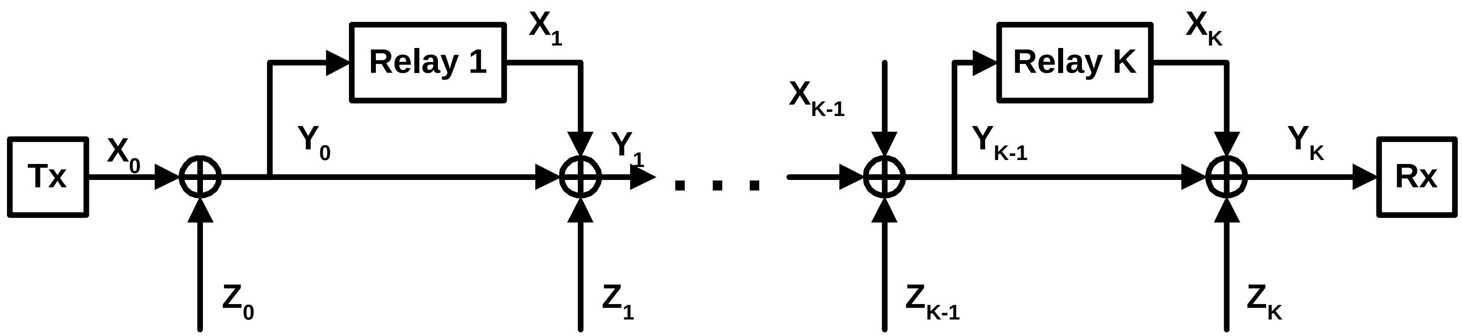
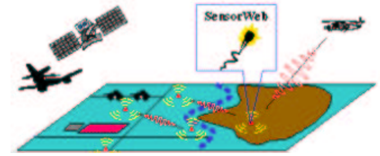


Figure 7: Broadcast Channel: Bandwidth factor penalty incurred by TDMA as a function of $R_1/R_2 = \theta$ for $\sigma_2^2 = 10\sigma_1^2$

Multirelay Channel





Theorem 8 *(Cover and El Gamal, 79) Single Relay:*

- *The capacity of this channel is given by*

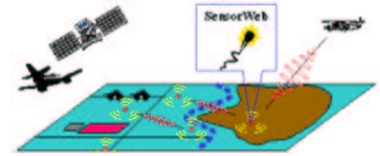
$$\mathcal{C}_1 = \max_{0 \le \alpha \le 1} \min \left\{ C\left(\frac{1+\beta_1+2\sqrt{(1-\alpha)\beta_1}}{1+\nu_1} \frac{P_0}{N_0} \right), C\left(\alpha \frac{P_0}{N_0} \right) \right\} \tag{12}$$

  where $C(x) = \log(1+x)$ $P_0 =$ *power at the transmitter,* $P_1 =$ *power at the relay;* $\beta_1 = P_1/P_0$ *and* $\nu_1 = \frac{N_1}{N_0}$.

- *let* α_1^* *denote the value of* α *which achieves the optimum in (12). Then*

$$\mathcal{C}_1 = C\left(\alpha_1^* \frac{P_0}{N_0} \right) = C\left(\alpha_1^* \frac{P_0(1+\nu_1)}{N_0+N_1} \right) \tag{13}$$

  where $\alpha_1^* = 1 \Leftrightarrow \beta_1 \ge \nu_1$



Theorem 9 *(Reznik, SK, SV, 2002) Multirelay capacity*

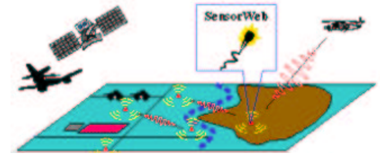
$$\mathbf{C}_{K-relay} = \max_{\{\alpha_{i,j}\}} \mathcal{C}_K \tag{14}$$

$$\mathcal{C}_K = \min_{0 \le k \le K} \mathcal{R}_k \tag{15}$$

$$\mathcal{R}_k = \sup C\left(P_0 \frac{\sum_{j=0}^{k} \left(\sum_{i=0}^{j} \sqrt{\alpha_{i,j}} \right)^2}{\sum_{j=0}^{k} N_j} \right) \tag{16}$$

the supremum is over the set of $\{\alpha_{i,j}\}$ *defined for* $0 \le i \le j \le K$ *satisfying the constraints*

$$\sum_{j=0}^{K} \alpha_{0,j} = 1 \qquad \sum_{j=i}^{K} \alpha_{i,j} = \beta_i = \frac{P_i}{P_0} \quad \forall 1 \le i \le K$$



Optimum Power Allocation

Maximize capacity by optimizing P_0, P_1, ... P_K subject to $P_0 + \cdots + P_K = P_T$.

$$\alpha_{i,k} = \alpha_k = \frac{1}{(k+1)^2}\nu_k \left(\sum_{j=0}^{k-1} (j+1)^2 \alpha_j \right) \quad \forall 0 \le i \le k \tag{17}$$

where

$$\nu_k = \frac{N_k}{\sum_{j=0}^{k-1} N_j} \tag{18}$$

Optimal power allocation is then given by:

$$\beta_k = \frac{\sum_{k=j}^{K} \alpha_j}{\sum_{j=0}^{K} \alpha_j} \tag{19}$$

Beamforming gain

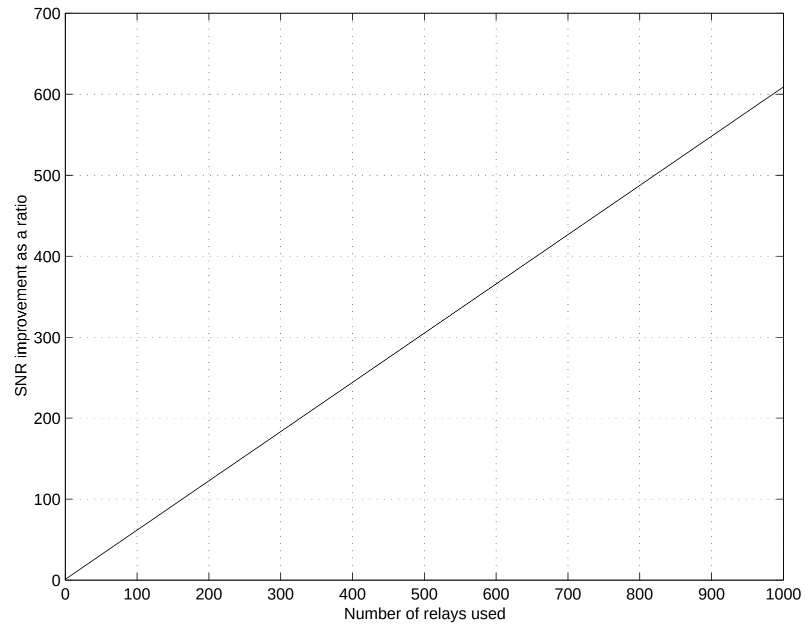
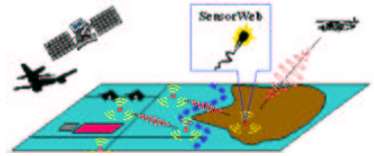


Figure 8: effective SNR (linear) / SNR pooling all powers noncoherently

Slope is (asymptotically) $= \frac{6}{\pi^2}K$, instead of K (with genie-aided relays that have noiseless access to message). Penalty = -2.16 dB.



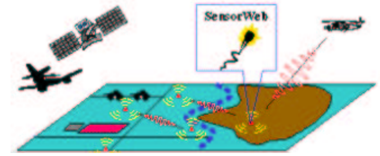
Delay Constraints and Causal Feedback

(Caire, Tuninetti, SV, 2001)

- K transmitters must deliver their message within N slots ($N \nrightarrow \infty$) to the receiver by spending a fixed maximum energy.
- The number of complex dimensions per slot is $L = \lfloor WT \rfloor$, where T is the slot duration and W is the channel bandwidth.
- The baseband complex received L-vector in slot n is

$$\boldsymbol{y}_n = \sum_{k=1}^{K} c_{k,n} \boldsymbol{x}_{k,n} + \boldsymbol{z}_n \tag{20}$$

where $\boldsymbol{z}_n$ is a proper complex Gaussian random vector with i.i.d. components of zero mean and unit variance, $c_{k,n}$ is the complex fading coefficient for user k with instantaneous power $\alpha_{k,n} \stackrel{\Delta}{=} |c_{k,n}|^2$

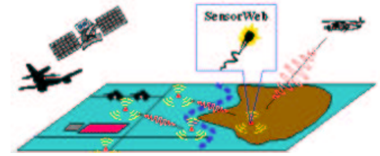


- The receiver has perfect Channel State Information (CSI)
- The transmitters have perfect *causal* feedback of CSI. (Major benefits of feedback in low power regime)
- Each transmitter k is subject to the per-codeword input constraint (referred to as *short-term* power constraint),

$$\frac{1}{L}\sum_{n=1}^{N}|\boldsymbol{x}_{k,n}|^2 \leq N\gamma_k \tag{21}$$

  where $N\gamma_k$ is the transmitted energy per L-symbols. γ_k has the meaning of average *transmit* Signal-to-Noise Ratio (SNR).
- The *instantaneous* transmit SNR of user k in slot n is

$$\beta_{k,n} \triangleq \frac{1}{L}|\boldsymbol{x}_{k,n}|^2 \tag{22}$$



Theorem 10 *The k-th user single-user long-term average capacity per unit energy $s_N^{(k)}$ is given by the Dynamic Programming recursion*

$$s_n^{(k)} = \mathrm{E}[\max\{s_{n-1}^{(k)}, \alpha_k\}] \tag{23}$$

for $n = 1, \ldots, N$ with initial condition $s_0^{(k)} = 0$ and where expectation is with respect to $\alpha_k \sim F_\alpha^{(k)}(x)$. Furthermore, $s_N^{(k)}$ is achieved by the "one-shot" power allocation policy defined by

$$\beta_{k,n}^\star = \begin{cases} N\gamma_k & \text{if } n = n_k^\star \\ 0 & \text{otherwise} \end{cases} \tag{24}$$

where the random variable $n_k^\star$, function of $(\alpha_{k,1}, \cdots, \alpha_{k,N})$, is defined as

$$n_k^\star = \min\left\{n \in \{1, \ldots, N\} : \alpha_{k,n} \geq s_{N-n}^{(k)}\right\} \tag{25}$$

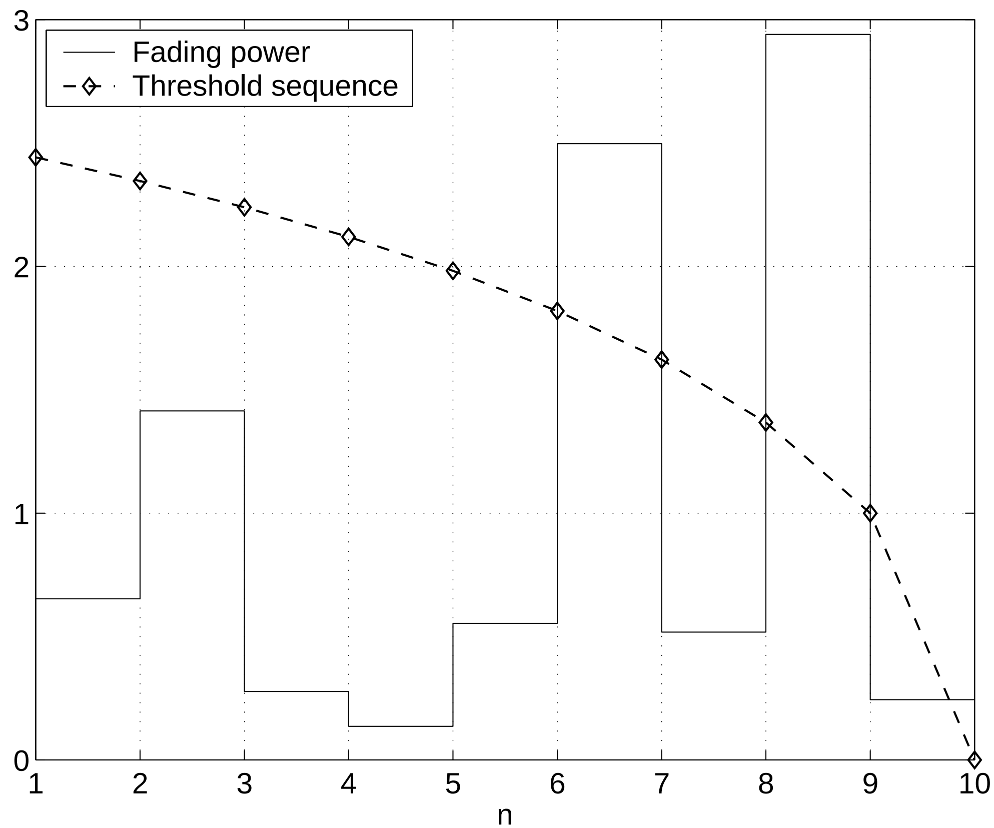


Figure 9: Rayleigh fading realization over a frame of $N = 10$ slots and the corresponding thresholds for the “one-shot” policy. Threshold function depends on Fading distribution.

Suboptimality of TDMA

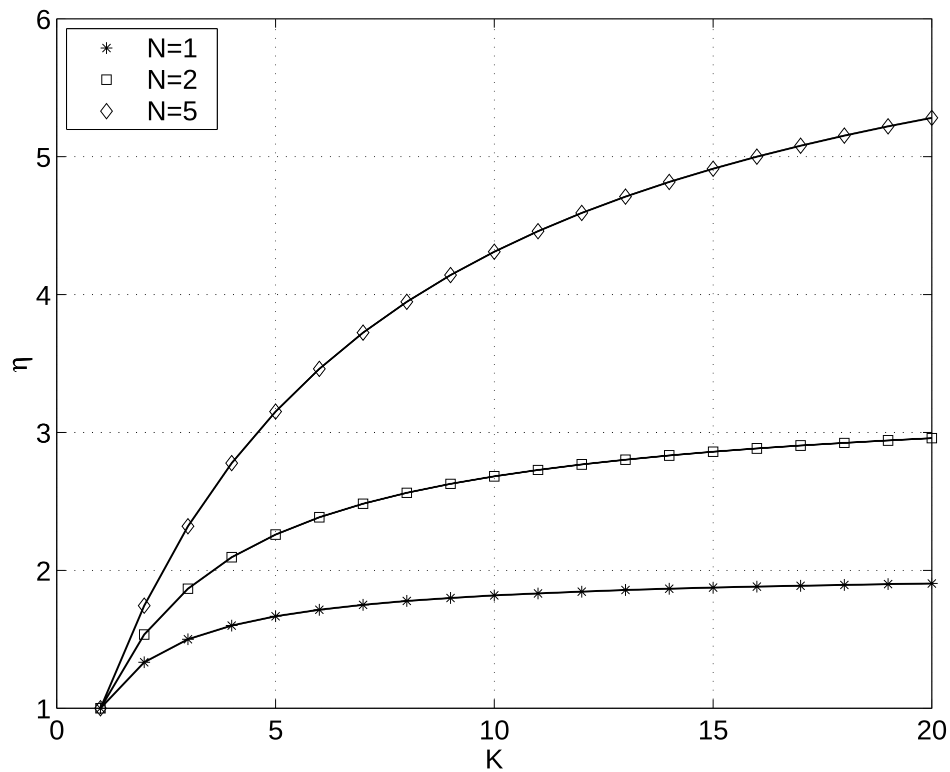
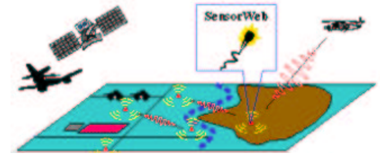
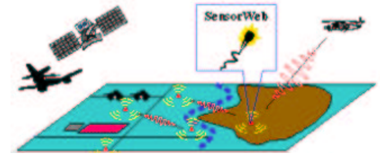


Figure 10: Bandwidth expansion factor of TDMA over superposition coding vs. the number of users K for the Rayleigh fading case.



- A population of sensors that have to transmit a measurement within a given delay by spending a fixed energy to a central collector terminal. (e.g. sensors for Earth observation deployed on a vast geographical area, and a low-Earth orbit satellite that illuminates them with its spotbeam antenna for a limited amount of time (say NT seconds) every orbit period.)
- Sensors have a battery that can be recharged during the orbit period, and use multiple resolution source coding in order to encode their measurement data with different rate-distortion tradeoffs.
- When the satellite is over the sensors, it broadcasts a probe signal in order to allow the sensors to measure their own channel



- The sensors spend the whole battery energy in a one-shot transmission at the instantaneous rate allowed by their fading state, according to the optimal power policy described here.
- TDMA (e.g. Bluetooth) is highly suboptimal in terms of bandwidth. Superposition (with receiver signal processing) is much more bandwidth efficient.